Nonlinear Integrable Optics Experiments Based on Symplectic Integrators

Stanislav Baturin

Department of Physics and
Department of Electrical Engineering
Northern Illinois University

Alexander Romanov, Alexander Valishev

Fermilab

06/17/2020

Outline

- Theoretical background
- Experimental proposal and simulations
- Summary

Theoretical background

Smooth distribution of the nonlinearity

What we start with:

$$H = \frac{p_x^2}{2} + \frac{p_y^2}{2} + K_2(s)\left(\frac{x^2}{2} + \frac{y^2}{2}\right) + V(x, y, s)$$

$$x_N = \frac{x}{\sqrt{\beta}} \qquad y_N = \frac{y}{\sqrt{\beta}}$$

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + \underbrace{\beta(s) V\left(x_N\sqrt{\beta(s)}, y_N\sqrt{\beta(s)}, \cancel{s}\right)}_{U(x_N, y_N)}$$

$$U(x_N, y_N) \sim \beta(s)\frac{\left(\sqrt{\beta(s)}x\right)^3}{3}\underbrace{K_3(s)}_{K_3(s) \sim \frac{K_3^{(0)}}{\beta(s)^{5/2}}}$$

What we want:

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + U(x_N, y_N)$$

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **13**, 084002 (2010)

Nonlinear accelerator lattices with one and two analytic invariants

V. Danilov

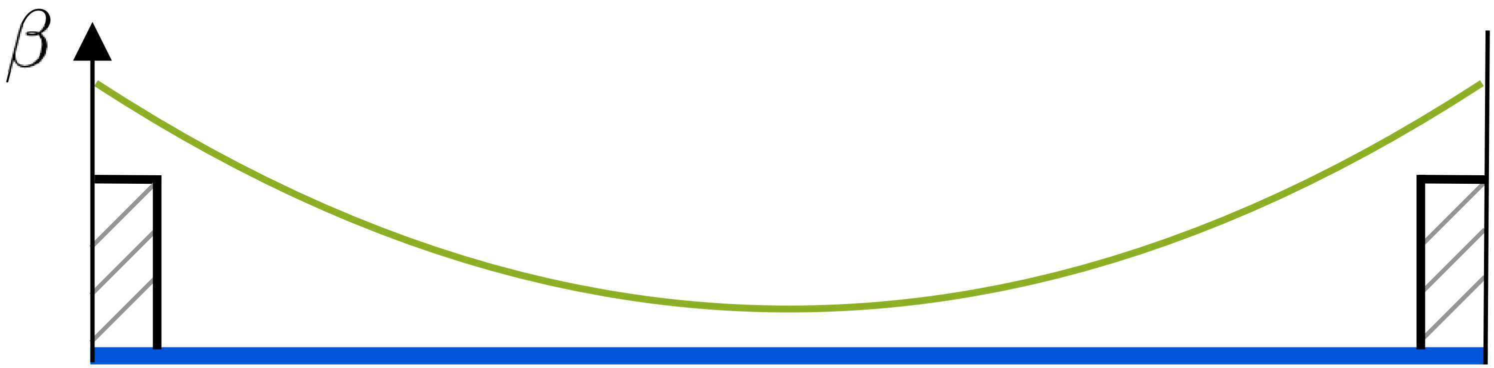
Spallation Neutron Source Project, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830, USA

S. Nagaitsev

Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA

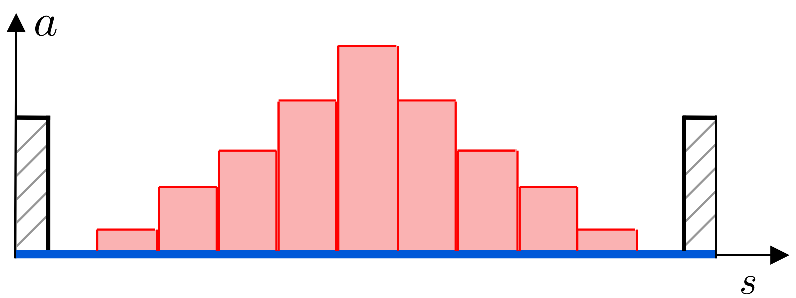
(Received 3 March 2010; published 25 August 2010)

T-insert. Linear optics design

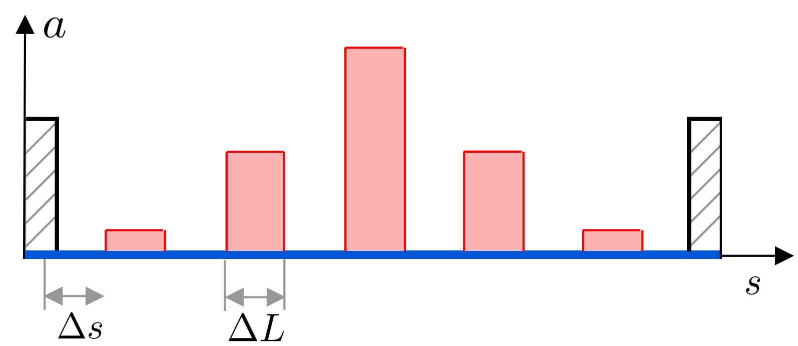


$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -k/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -k/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -k/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -k/2 & 1 \end{bmatrix}$$

Nonlinear magnets placement



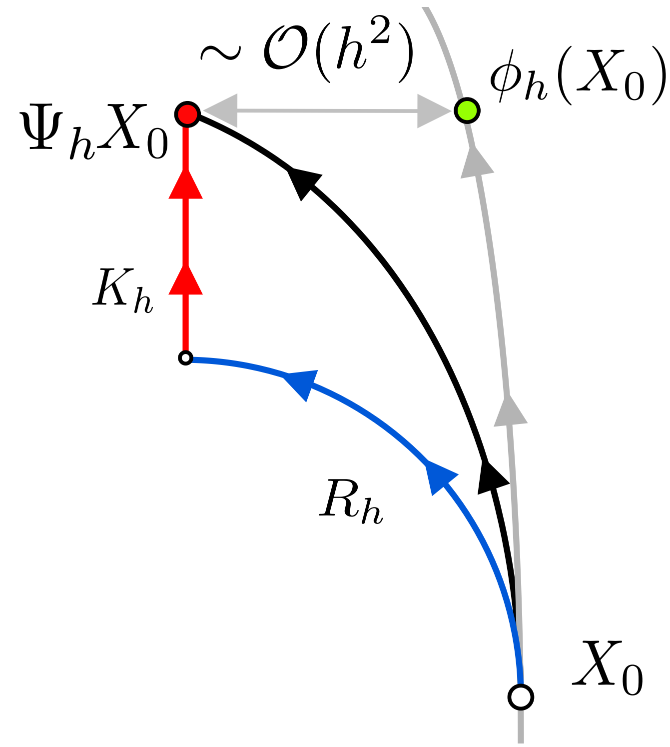
$$\beta(s) V\left(x_N\sqrt{\beta(s)}, y_N\sqrt{\beta(s)}, \cancel{s}\right)$$



Alternative discretization strategy

$$\mathrm{H} = \underbrace{\frac{q_1^2 + p_1^2}{2} + \frac{q_2^2 + p_2^2}{2}}_{} + \underbrace{U(q_1, q_2)}_{}$$

$$\Psi_h = R_h \circ K_h$$

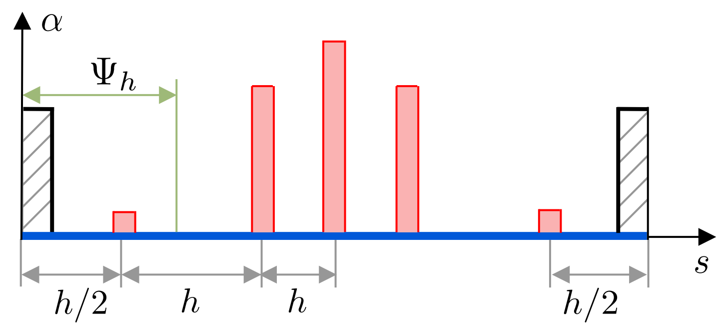


Algorithm

1 . Pick an integrator and a step in phase

$$\Psi_h$$

2. Set optics with the step in phase prescribed by the integrator (higher orders integrators may have non constant step)



3. Set nonlinear magnet strength according to similarity transformation with betatron amplitude matrix

$$\mathbf{B}(s_1) \circ K_h \circ \mathbf{B}^{-1}(s_1)$$

Connection to the lattice

$$M_{x,y}(s_2|s_1) = \mathbf{B}_{x,y}(s_2) R_{\phi_{x,y}} \mathbf{B}^{-1}_{x,y}(s_1)$$

$$\mathbf{B}_{x,y}(s) = \begin{bmatrix} \sqrt{\beta_{x,y}(s)} & 0 \\ -\frac{\alpha_{x,y}(s)}{\sqrt{\beta_{x,y}(s)}} & \frac{1}{\sqrt{\beta_{x,y}(s)}} \end{bmatrix}$$

Connection to the lattice

$$X_h = R_h \circ K_h X_0$$

$$\mathrm{X}_h = M(s_1|s_0) \circ \mathbf{B}(s_0) \circ K_h \circ \mathbf{B}^{-1}(s_0)\mathrm{X}_0$$

Transformation of the nonlinear kick

$$\mathrm{X}_{h,0} \equiv \mathbf{B} X_{h,0}$$

Connection to the lattice

Step in phase

$$\mathbf{B}(s_1) \circ K_h \circ \mathbf{B}^{-1}(s_1)\mathrm{X}_0 = \left[x^0, P_x^0 - h\partial_x U\left(\frac{x}{\sqrt{\beta_x}}, \frac{y}{\sqrt{\beta_y}}\right), y^0, P_y^0 - h\partial_y U\left(\frac{x}{\sqrt{\beta_x}}, \frac{y}{\sqrt{\beta_y}}\right)\right]^{\mathrm{T}}$$

Transformed nonlinear kick

$$\exp\left(-\delta L :\mathrm{V}:\right) X_0 = \left[x^0, P_x^0 - \delta L\, \partial_x V\left(x, y, s\right), y^0, P_y^0 - \delta L\, \partial_y V\left(x, y, s\right)\right]^{\mathrm{T}}$$

Flow of the thin magnet

Length of the magnet

Connection to the lattice

Constant!

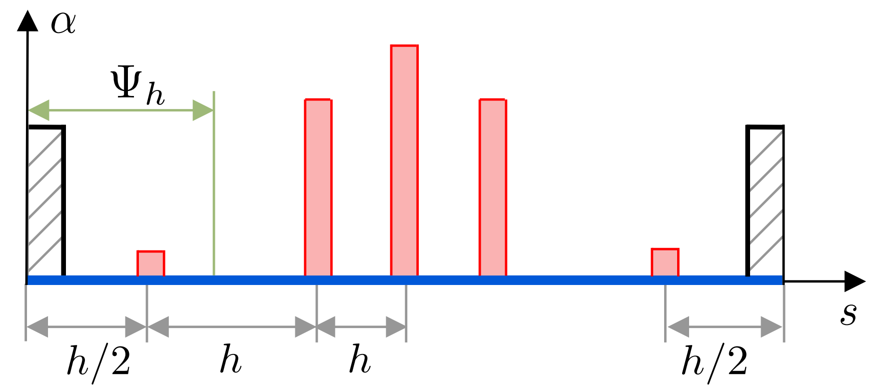
$$\boxed{V(x,y,s) = \frac{h}{\delta L} U\left(\frac{x}{\sqrt{\beta_x(s)}}, \frac{y}{\sqrt{\beta_y(s)}}\right)}$$

$$\boxed{\begin{array}{l} h \to 0 \\ \delta L \to 0 \end{array}} \Rightarrow \boxed{\frac{h}{\delta L} = \frac{d\phi}{ds} = \frac{1}{\beta}}$$

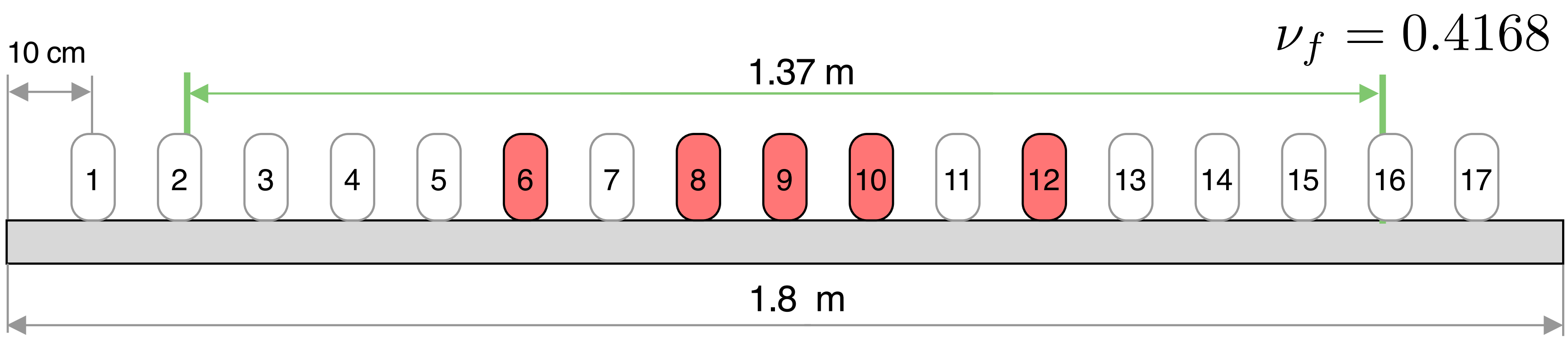
Experimental proposal and simulations

Ruth lattice

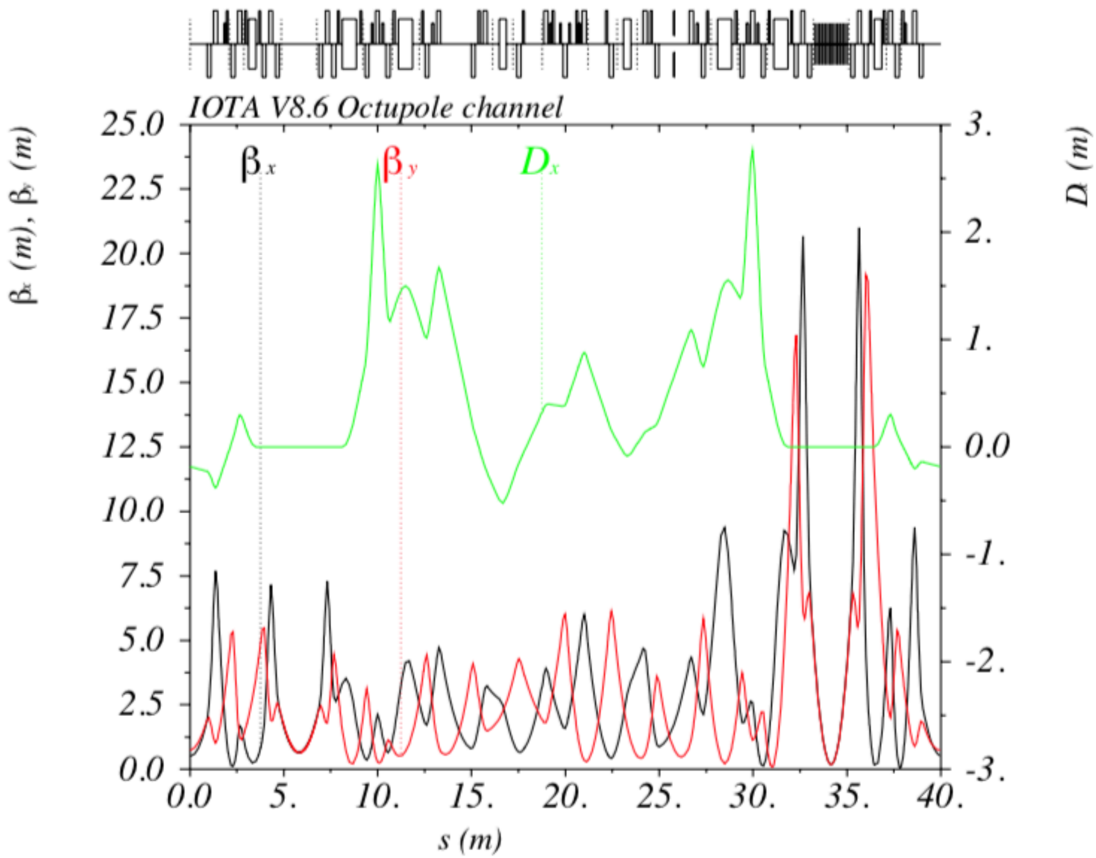
$$\Phi_h = R_{h/2} \circ K_h \circ R_{h/2}$$



Octupoles arrangement (equal phase)



Ruth lattice (IOTA Run-2 configuration)



- Note that in order to implement the tune of 0.41, the mirror symmetry is ruined.
- The large phase advance through nonlinear drift creates a 'final-focus-like' optics with large betas and high chromaticity

Ruth lattice. Simulations.

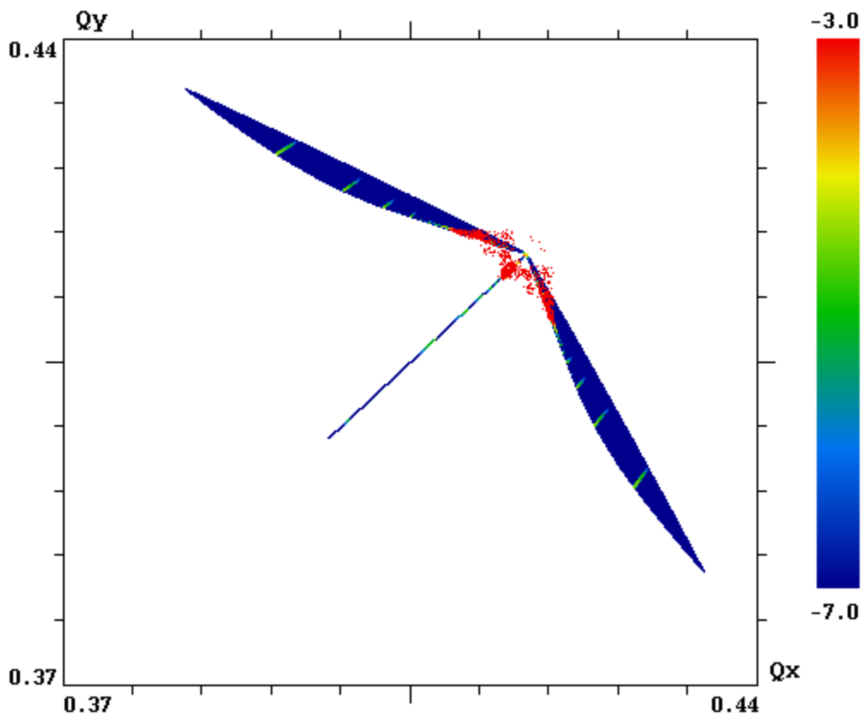
Simple model

- The model of the machine arc is a map characterized by
  - Beta and alpha functions at entrance and exit
  - Dispersion at entrance and exit
  - Phase advances (x, y, z)
  - Chromaticity
- Octupole channel is modeled as five thin multipoles, separated by drifts

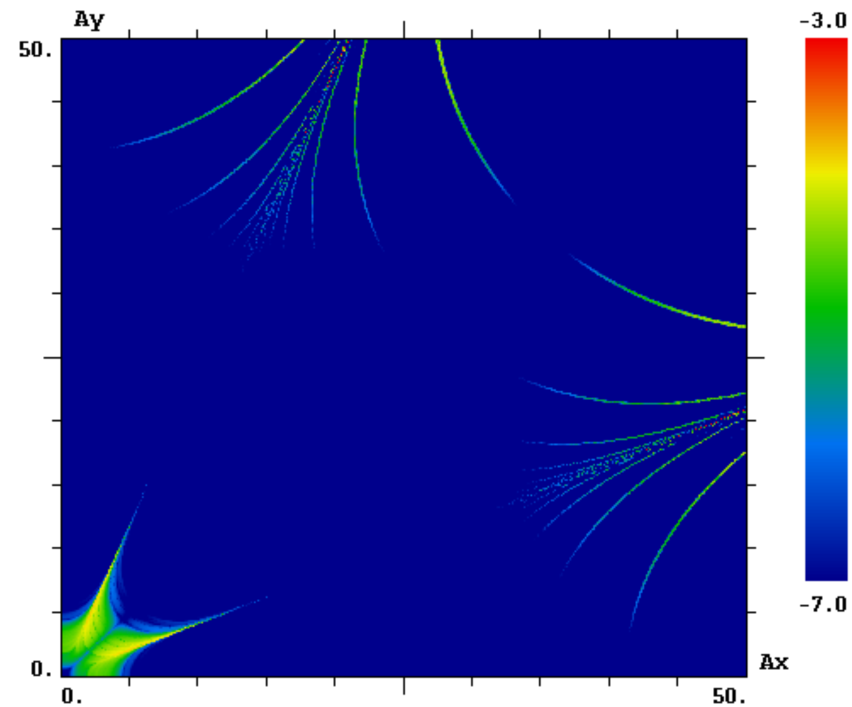
Full lattice model

- Thin element tracking

Simple model: ideal case

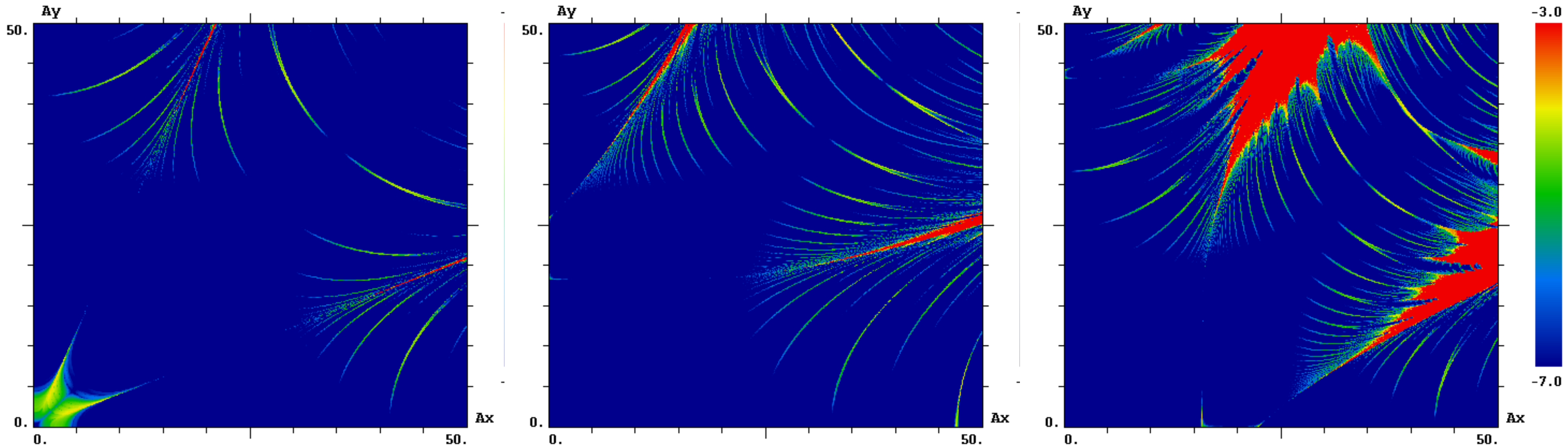


Tune space. Q_0=0.41



Amplitude space.
A=50 corresponds to beam pipe aperture in IOTA

Simple model with imperfections

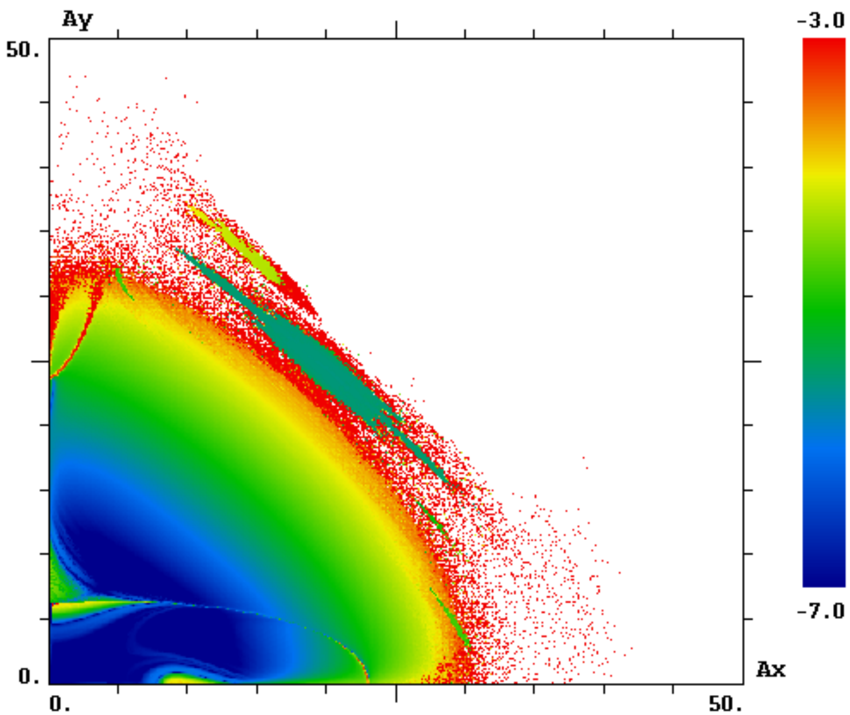


Case 1:
2% random error of octupole gradient

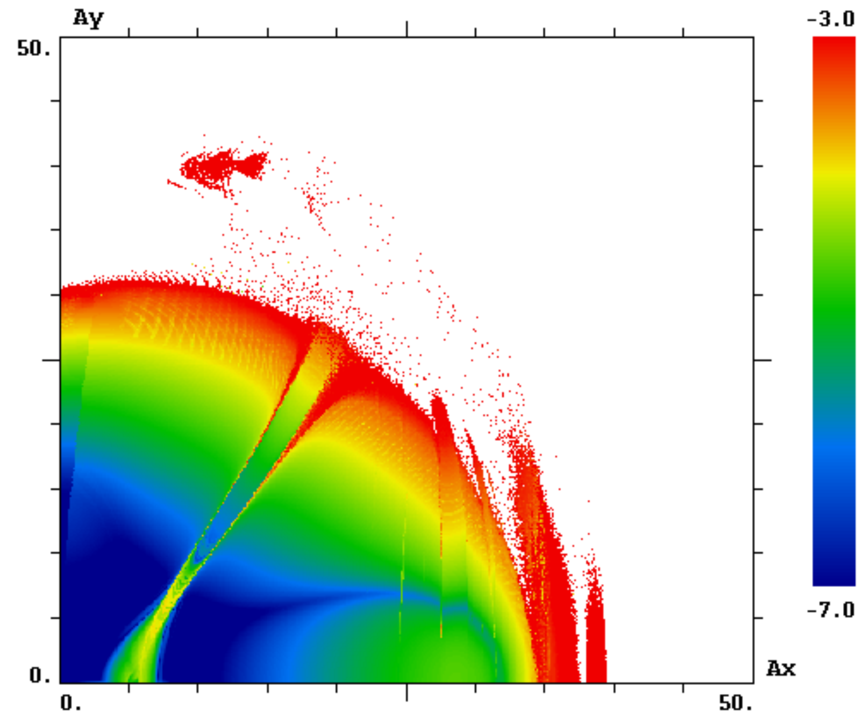
Case 2:
Same as 1 +
0.01 error of phase advance
through accelerator arc

Case 3:
Same as 2 +
5% error of beta-function
of accelerator arc

Full lattice simulations



Octupoles off
Corrected chromaticity (Ch=Cv=0)



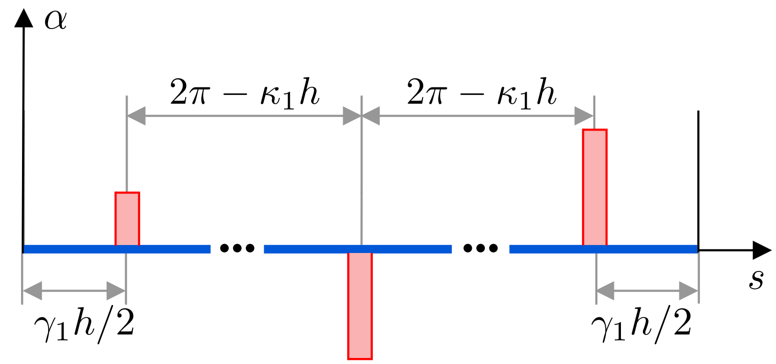
Octupoles on
DA slightly improves, still dominated by sextupoles

Yoshida lattice

Yoshida forth order method

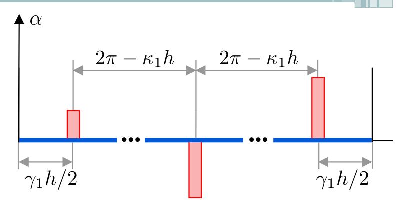
$$\Phi_h^Y = \Phi_{\gamma_3 h} \circ \Phi_{\gamma_2 h} \circ \Phi_{\gamma_1 h},$$

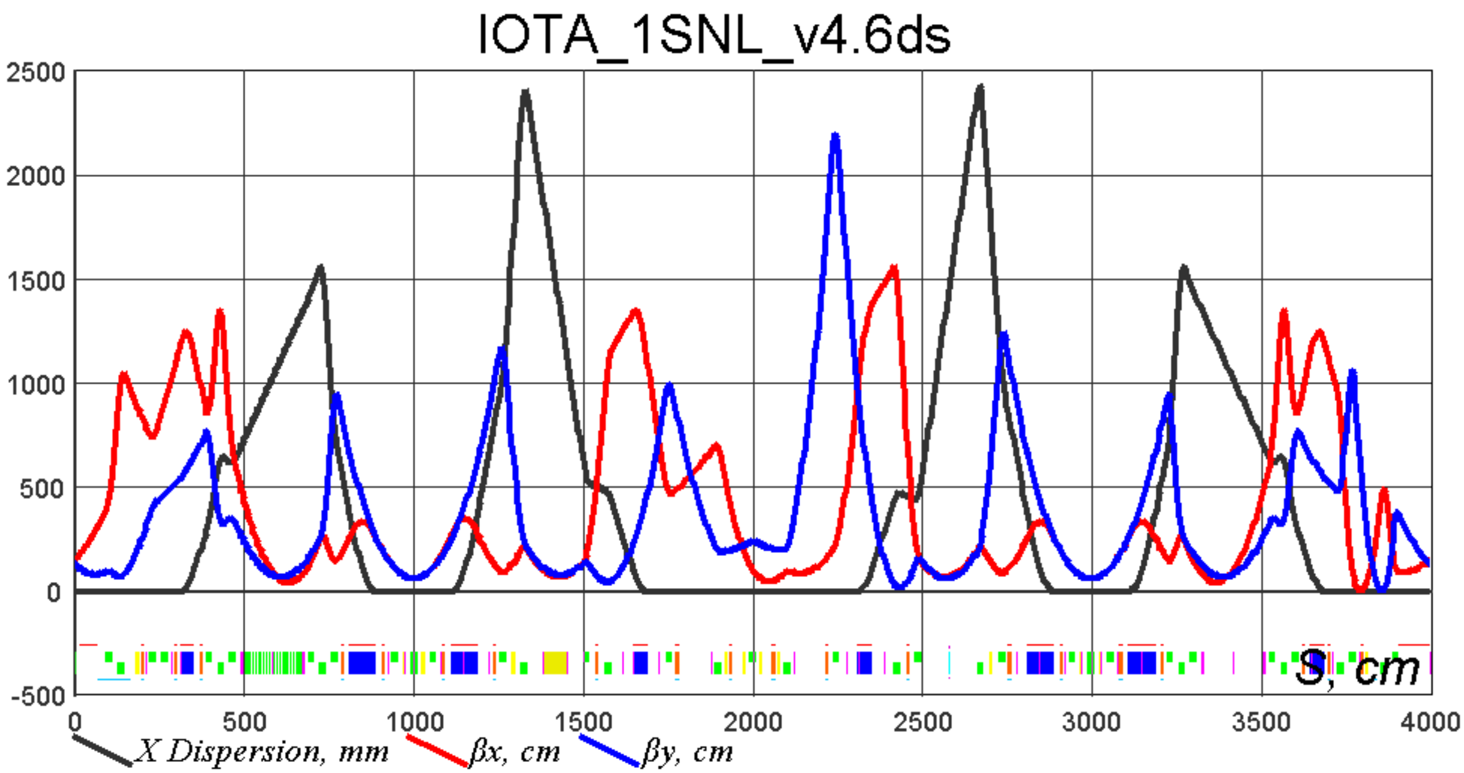
$$\gamma_1 = \gamma_3 = \frac{1}{2 - 2^{1/3}}, \quad \gamma_2 = -\frac{2^{1/3}}{2 - 2^{1/3}}$$



- Implement it with 3 of 4 IOTA sextupoles

Yoshida lattice (IOTA Run-2 configuration)





Simulations summary

- Ruth lattice could be implemented with 5 octupoles with realistic imperfections and will produce strongly nonlinear system
  - Strong chromaticity, if uncorrected, complicates the dynamics
  - The strength of IOTA octupole magnets limits the attainable tune shift to 0.02
  - Chromaticity correction with the sextupoles available in Run-2 (four magnets SC1R, SC2R, SC1L, SC2L) limits dynamical aperture but still allows reasonable measurements
- Yoshida lattice is feasible for IOTA.
  - Only preliminary simplified tracking was performed.
  - Error analysis and additional simulations are in progress.

Measurables

- Tune footprint
- 4D Hamiltonian
- Potentially real Poincare surface of section with synclight and BPM. (x,y for example with one of the transverse momentums fixed)

Summary

- Nonlinear integrable lattice can be implemented with just few magnets and can be tested at IOTA.

- Ruth lattice potentially can be implemented in Run-3 as octupoles remain in place. New optics need to be developed and commissioned.

- Yoshida lattice could be tested in Run-4.

- Both lattices require minimum machine modification.

Thank you!